Int. J. Heat Mass Transfer. Vol. 30, No. 10, pp. 2193–2195, 1987
Printed in Great Britain

0017–9310/87 \$3.00+0.00
Pergamon Journals Ltd.

Convection heat transfer to the continuous cylinder of finite thermal capacity moving inside the heating pipe

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(*Received* 2 *November* 1986)

1. INTRODUCTION

THE PROBLEM of convective heat transfer to the continuous cylinder moving through a heated pipe is of great industrial importance. One of the most frequent cases when this kind of heating is used for the technological process of so-called texturization (bulking) of synthetic fibres is solved here.

The main process consists of imparting a false twist to a yarn consisting of individual filaments while it is heated in a pipe. The yarn is thereafter cooled and the previously imparted crimp is reset thus producing three-dimensional loops and yarn bulk.

The solution of a similar problem was not found in the literature, even though in many papers [2–5] the heat transfer from yarn to free space is described and in ref. [6] and a different problem with a similar initial condition is solved.

The object of this note is to present an analytical solution of the equation of heat transfer from the heated pipe to the continuous cylinder moving by the axis of cylinder, for $Re < 300$ when the creation of the thermal and velocity boundary layers in the whole pipe can be supposed. The coefficient of heat transfer is determined from the calculated temperature function and the results are compared with experimental values.

2. FORMULATION OF THE PROBLEM (Fig. 1)

The twisted chemical yarn considered as a continuous cylinder of constant radius r_c of constant temperature in the whole cross-section and finite thermal heat capacity is moving without vibrations with velocity v_c along the axis of the straight heating isothermal pipe of inner radius R. The cavities between individual filaments of the fibre are characterized by the porosity coefficient $\beta > 1$, the thermal properties of the fibre can be characterized by the average thermal capacity ρc. The thermal properties of the air in the pipe are supposed to be constant. The stationary temperature field in the pipe neglecting the dissipation heat, the axial part of heat transfer, the radiative heat transfer and the rotation of the yarn (the negligible influence of rotation of the yarn was confirmed experimentally [1]), is described by

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}-\frac{v(r)}{a}\frac{\partial\theta}{\partial z}=0 \tag{1}$$

where the function $\theta(z, r)$ is the difference between the temperature of the pipe wall and the temperature at (z, r) inside the pipe, a is the temperature conductivity of the air and $v(r)$ is the velocity distribution in the pipe. Together with equation (1) the following boundary conditions must be satisfied:

$$\frac{\partial\theta}{\partial r}=C\cdot\frac{\partial\theta}{\partial z} \tag{2}$$

where $C = r_c c\rho v_c\beta/2\lambda$ at the surface of the fibre, that is for $r = r_c$ (it expresses the equality of the heat penetration through the boundary layer to the surface of the yarn and the heat accumulated by the yarn considering its finite thermal capacity), and for

$$\theta(z, R) = 0 \tag{3}$$

at the pipe wall. Besides this the yarn and the air at the beginning of the pipe are supposed to be of constant temperature

$$\theta(0, r) = \theta_0 \quad \text{for} \quad r_c \leqslant r < R \tag{4}$$

and obviously

$$\theta(\infty, r) = 0. \tag{5}$$

The velocity distribution calculated in ref. [1]

$$v(r)=\frac{g\beta\theta}{4v}(R^2-r^2)+\left(v_c+\frac{g\beta\theta}{4v}(R^2-r_c^2)\right)\cdot\frac{\ln r-\ln R}{\ln r_c-\ln R}$$

was replaced by the approximate velocity distribution

$$v(r)=v_c\left(\frac{r}{r_c}\right)^m \tag{6}$$

enabling the transfer of the calculation of the temperature function to the solution of the Bessel equation. Both distributions satisfy the same initial condition $v(r_c) = v_c$ and yield the same average velocities.

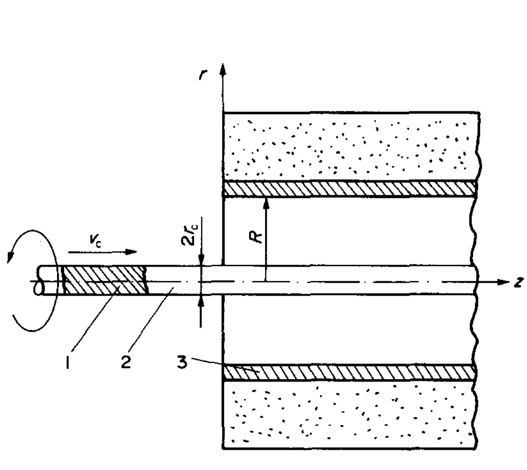

FIG. 1.

3. EXACT SOLUTION

Particular integrals of equation (1) are anticipated to be of the form

$$\theta_n(z, r) = T_n(r)\,e^{-\alpha_n^2 z} \tag{7}$$

which automatically satisfies the physical requirement (5). Then $T_n(r)$ fulfils the ordinary differential equation

$$\frac{d^2T_n(r)}{dr^2}+\frac{1}{r}\frac{dT_n(r)}{dr}+\frac{v(r)}{a}\alpha_n^2T_n(r)=0 \tag{8}$$

NOMENCLATURE

a	temperature conductivity (diffusivity)
A_n	coefficients of the temperature function expansion (18)
b	constant (parameter) in equation (14)
c	specific heat
C	constant in equation (2)
$f_n(x)$	transformed radial function
g	gravity (acceleration of gravity)
J_0, J_1	zero- and first-order Bessel functions of the first kind
K	constant (parameter) in equation (13)
l	length of pipe
m	exponent in the velocity function
r_c	radius of the cylinder (fibre)
R	radius of the pipe
Re	Reynolds number
$T_n(r)$	radial part of the temperature function
v_c	speed (velocity) of the fibre along the z-axis
$v(r)$	velocity function (the velocity distribution) in the pipe
x	transformed radial coordinate
Y_0, Y_1	zero- and first-order Bessel functions of the second kind
z, r	cylindrical coordinate (z-axis is identical with the common axis of the cylinder and the pipe).

Greek symbols

α	coefficient of heat transfer
α_n	parameter in equation (8)
β	coefficient of porosity
λ	thermal conductivity
$\theta(z, r)$	temperature difference
ν	kinematic viscosity
ρ	density of the fibre polymer
ξ_n	eigenvalues of equation (11).

with boundary conditions

$$\frac{dT_n(r)}{dr} = -C\alpha_n^2 T_n(r) \quad \text{for} \quad r = r_c \tag{9}$$

$$T_n(R) = 0. \tag{10}$$

Introducing the substitution $x = (r/r_c)^{(m+2)/2}$, equation (8) turns into the Bessel equation for the function $f_n(x) = T_n(r)$

$$\frac{d^2 f_n(x)}{dx^2} + \frac{1}{x}\frac{df_n(x)}{dx} + \xi_n^2 f_n(x) = 0 \tag{11}$$

where

$$\xi_n = \frac{2x_c\alpha_n}{m+2}\sqrt{\left(\frac{v_c}{a}\right)} \tag{12}$$

and boundary conditions (9) and (10) could be transformed to

$$\frac{df_n}{dx} = -\xi_n^2 K f_n \quad \text{for} \quad x = 1 \tag{13}$$

$$f_n(b) = 0, \quad b = \left(\frac{R}{r_c}\right)^{(m+2)/2} \tag{14}$$

where

$$K = \frac{c\rho\beta a(m+2)}{4\lambda}. \tag{15}$$

The general solution of equation (11) is the linear combination of the Bessel functions of the first and second kinds $J_0(\xi_n x)$, $Y_0(\xi_n x)$ the coefficients of which are determined from boundary condition (14). We obtain

$$f_n(x) = J_0(b\xi_n)Y_0(\xi_n x) - Y_0(b\xi_n)J_0(\xi_n x). \tag{16}$$

Substituting this relation into equation (13), we obtain the transcendental equation

$$Y_0(b\xi_n)J_1(\xi_n) - J_0(b\xi_n)Y_1(\xi_n) + \xi_n K[J_0(b\xi_n)Y_0(\xi_n) - Y_0(b\xi_n)J_0(\xi_n)] = 0 \tag{17}$$

for the eigenvalues ξ_n. The general temperature function which solves equation (1) and satisfies proper conditions then becomes

$$\theta(z,r) = \theta_0 \sum_{n=1}^{\infty} A_n f_n(x)\, e^{-\alpha_n^2 z}. \tag{18}$$

The determination of coefficients A_n would be easy in the case of the orthogonality of eigenfunctions $f_n(x)$. However, it shows they are not orthogonal in the usual sense of the word since it holds that

$$\int_1^b x f_m(x) f_n(x) = \frac{1}{\xi_n^2 - \xi_m^2}\left[\frac{df_n}{dx}f_m - \frac{df_m}{dx}f_n\right]\Bigg|_{x=1}. \tag{19}$$

This difficulty caused by the inhomogeneous boundary condition (13) can be removed by using so-called generalized orthogonal properties [7].

Substituting for the derivatives $df_n(x)/dx$ and $df_m(x)/dx$ in equation (19) from equation (13), we have

$$\int_1^b x f_n(x) f_m(x)\, dx + K f_n(1) f_m(1) = 0 \quad \text{for} \quad n \neq m. \tag{20}$$

Using initial condition (4) together with equation (18) gives

$$\sum_{n=1}^{\infty} A_n f_n(x) = 1 \quad \text{for} \quad 1 < x < b \tag{21}$$

$$\sum_{n=1}^{\infty} A_n f_n(1) = 1. \tag{22}$$

Multiplying equation (21) by $x f_m(x)\, dx$ and integrating over the interval $(1, b)$ and multiplying equation (22) by $K f_m(1)$ and adding both results, we obtain

$$\int_1^b x f_m(x) + K f_m(1) = \sum_{n=1}^{\infty} A_n \left[\int_1^b x f_n(x) f_m(x)\, dx + K f_n(1) f_m(1)\right]. \tag{23}$$

According to equation (20) the only non-vanishing term in the series on the right-hand side of equation (23) is that for which $n = m$, so that

$$A_n = \frac{\int_1^b x f_n(x)\, dx + K f_n(1)}{\int_1^b x f_n^2(x)\, dx + K f_n^2(1)}. \tag{24}$$

After the integration using boundary condition (17) and the Wronskian relation [8]

$$J_1(x)Y_0(x) - J_0(x)Y_1(x) = \frac{2}{\pi x} \tag{25}$$

Table 1

Case	$r_c \times 10^5$ (m)	$R \times 10^3$ (m)	v_c (m s^{-1})	ξ_1	ξ_2	ξ_3
1	5.10	5.00	1	0.01411	0.04734	0.09935
2	5.10	5.00	2	0.01558	0.06245	0.13142
3	5.10	5.00	3	0.01672	0.07143	0.14997
4	7.35	5.00	2	0.01877	0.09141	0.19162
5	7.35	3.25	2	0.02159	0.18412	0.38370
6	7.35	2.00	2	0.02532	0.36008	0.74304

we obtain

$$A_n = \frac{4\pi}{[\pi^2 \xi_n^2 f_n^2(1)(1+\xi_n^2 K^2 - 2K) - 4]}. \tag{26}$$

The final form of the temperature function is

$$\theta(z,r) = \theta_0 \sum_{n=1}^{\infty} \frac{4\pi f_n \left(\left(\frac{r}{r_c} \right)^{(m+2)/2} \right) e^{-\alpha_n^2 z}}{\pi^2 \xi_n^2 f_n^2(1)(1+\xi_n^2 K^2 - 2K) - 4} \tag{27}$$

and at the surface of the cylinder

$$\theta(z,r_c) = \theta_0 \sum_{n=1}^{\infty} \frac{4\pi f_n(1)\, e^{-\alpha_n^2 z}}{\pi^2 \xi_n^2 f_n^2(1)(1+\xi_n^2 K^2 - 2K) - 4}. \tag{28}$$

4. NUMERICAL RESULTS

The eigenvalues ξ_n were obtained numerically by Newton's method for six different cases differing from each other by the radius of the pipe and the yarn and by the speed of the yarn. In each of these six cases three eigenvalues were determined. These are listed in Table 1.

It has been found that series (18) converges very rapidly for $z > 0.2$ so that the temperature function is determined with sufficient accuracy (for our purposes) by the first term of the series.

As an illustration the values of the temperature function for case (4), $0.1 < z \leqslant 1$, $r_c < r < R$ were calculated. This is illustrated graphically in Fig. 2.

From the value of the temperature function at the surface of the fibre on the upper end of the pipe the coefficient of heat transfer α_{cal} was calculated by using the relation [1]

$$\alpha = -\frac{1}{2l} \rho r_c c \beta v_c \ln \theta(l, r_c). \tag{29}$$

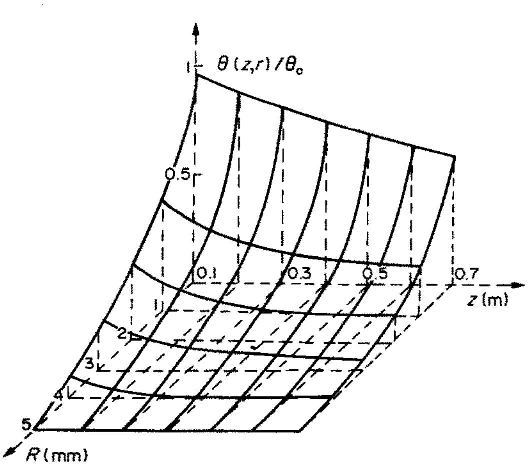

FIG. 2.

Table 2

Case	α_{cal} (W m^{-2} K^{-1})	α_{exp} (W m^{-2} K^{-1})	$\alpha_{cal}/\alpha_{exp}$
1	130.6	110.8	1.18
2	124.8	126.5	0.99
3	115.6	144.7	0.80
4	92.4	98.3	0.94
5	115.0	127.4	0.90
6	137.0	206.0	0.67

The heat transfer to the yarn from the pipe wall was realized experimentally for the parameters mentioned in Table 1. The values of the heat transfer coefficients were calculated by relation (29) from the output temperatures taken from the original convection thermometer [1].

The part belonging to the radiative heat transfer ($\alpha_{rad} = 14.7$–16.7 W m^{-2} K^{-1} [1]) was subtracted from the α-values obtained in this way and compared with theoretically calculated values. The results are summarized in Table 2.

From Table 2 it is evident that the differences between the theoretical and experimental values of α extend from -20 to 20%, which can be explained namely by replacing the real velocity profile by an approximate one except in the last case, where the influence of the turbulent heat transfer is apparent.

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